

## Analysis of Circular Reinforced Tunnels by Analytical Approach

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### Abstract

This paper addresses the problem of quantifying the mechanical contribution of rockbolts installed systematically around tunnels (regularly spaced around the tunnel) excavated in rock masses. Assuming that the mechanical contribution of grouted rockbolts is that of increasing internal pressure within a broken rock mass, a new procedure for computation of ground response curves for a circular tunnel excavated in strain softening material and then reinforced with systematically active grouted rockbolts is presented. In this work, the equation of the ground response curve for a tunnel which has been reinforced with passive grouted rockbolts is also derived. The proposed model allows one to take into account the effect of the distance of the bolted section to the tunnel face, the effect of increasing rockbolts spacing, the influence of increasing pretension load in calculating of the ground response curve, and the effect of increasing the cross-section area of rockbolts. The results show that decreasing rockbolts spacing increases the support system stiffness rather than preloading of them.

**Keywords:** Analytical solution; Active grouted rockbolts; Tunnel design; Convergence-confinement; Reinforced tunnel

### 1. Introduction

#### 1.1. General Definitions

Rockbolts are among the most popular systems of support in tunneling operations. Speed in installation, effectiveness in different rock mass quality especially in weak rock masses, flexible density in using them while varying geotechnical condition (NATM method), minimum installation space and cost, are many factors which have contributed to the increasing acceptance of the bolting as a favorable support system in underground structure stabilization.

Rockbolt reinforcement has been used as a means of stabilizing civil engineering tunnels for the last 60 years. Although the method of reinforcing rock masses with steel bars has been applied in mining works since the late 19<sup>th</sup> century, the turning point in the rational design and application of rockbolts can be traced back to the experience made during design and construction of underground caverns for the Snowy Mountain hydroelectric scheme in Australia [1, 2]. Then, Lang [3, 4] introduced the concept holding that rockbolts can be used

to 'lock together' blocks in heavily jointed rock. In order to demonstrate the 'locking' effect provided by rockbolts, Lang [3] also conducted a classical experiment in which a bucket was filled with coarse gravel and threaded metal rods are inserted and tensioned to act as rockbolts within the gravel (Fig 1).

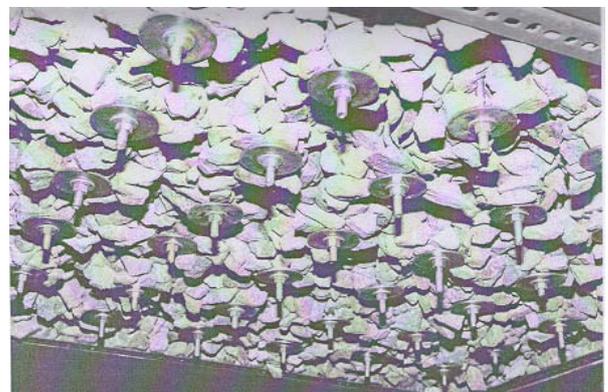


Fig. 1. Locking of coarse gravel by still bars [3]

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Different types of rockbolts are used in design and construction of tunnels. Hoek and Brown [5] and Stillborg [6] provide detailed descriptions and illustrations of different rockbolt configurations used in practice. From a simplistic mechanical point of view, rockbolts can be divided into the two basic categories represented in Fig. 2: a) grouted rockbolts and b) anchored rockbolts [7]. In a grouted rockbolt, the shank of the rockbolt is placed into a drill hole and cement grout is injected in the space between the shank and the wall of the hole. As represented in Fig. 2a, a face plate is then secured at the head of the rockbolt with a washer and nut. The vector  $F_r$  in Fig. 2a represents the total force transferred from the rock to the rockbolt at the head of the rockbolt through shear stresses, indicated by  $\tau_g$  vectors along the shank of the rockbolt. In an anchored rockbolt (Fig. 2b), the shank of the rockbolt is placed into a drill hole with an expansion shell attached to the foot of the rockbolt. The vector  $F_r$  in Fig. 2b represents the total force transferred from the rock to the rockbolt at the head of the rockbolt while the vector  $F_b$  represents the reaction developed at the foot of the rockbolt. In an anchored rockbolt (Fig. 2b), the axial force along the shank of the rockbolt is constant and therefore  $F_r = F_b$ .

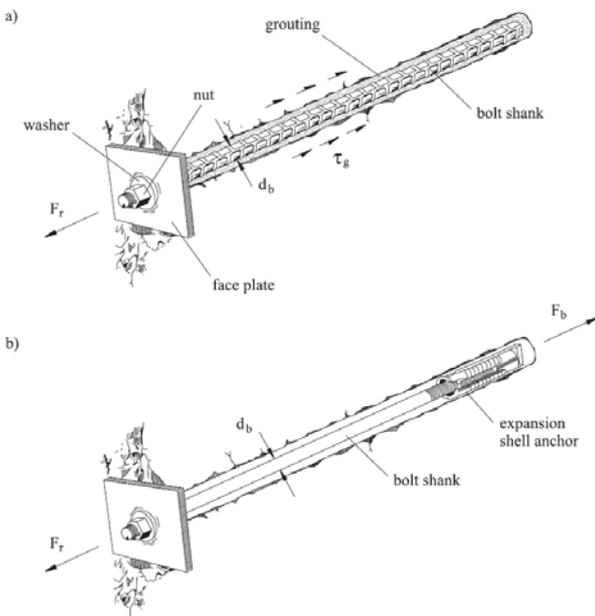


Fig. 2. (a) Grouted and (b) Anchored rockbolts [6]

The grouted and anchored rockbolts are sometimes installed under tension (pre-tensioned or active rockbolt). In the grouted type, the cement grout is injected at the end part of along the bolt; then, it is tensioned and the head of the rockbolt is tied with a washer and nut to face plate. At last, the remainder part of the bolt is grouted. The purpose of pre-tensioning the rockbolts is to have them transfer

part of this initial (tensile) load as an active compressive load that increases further the resulting stress confinement in the rock mass.

It should be noted that there is no difference between passive and active grouted rockbolts behavior mechanism after installation in tunnels. But, effectiveness of active types is more than passive types due to pre-loading of them.

### 1.2. Tunnel Support System with Grouted Rockbolts

Systematic rockbolting is nowadays a standard practice in design and construction of tunnels in rock and a key component in technologies used for designing tunnels, such as New Austrian Tunneling Method [8]. In this method the purpose of rockbolting are used to lock together and prevent detachment of individual blocks.

Systematic rockbolt reinforcement in tunneling is normally designed through application of empirical, numerical and analytical approaches. The empirical approaches, the Q-System [9] and the RMR-System [10] provide guidelines on density of rockbolt reinforcement required to stabilize underground openings as a function of dimension of the opening and the quality of the rock mass. Numerical approaches based on the finite element, boundary element or distinct elements methods, have been widely applied in recent years due to the possibility of taking constitutive laws especially designed for geological materials, complex geometry and a good simulation of the step by step tunneling operations, into account. Analytical approaches are used to identify critical parameters; they can provide a quick estimate of the rock and reinforcement behavior, and can be used for preliminary design analyses or for pre dimensioning of the reinforcement.

The use of analytical approaches has encountered many theoretical problems as it is difficult to model the properties of reinforced material with the usual proposed models. This is because, unlike the other support systems, the grouted rockbolts do not act independently of the rock mass and hence the deformations, which occur in both the rock mass and the support system, cannot be separated [5].

According to the previous studies, the grouted rock bolting effects are summarized as the ground sewing, adding pressure on the tunnel boundary [11], cohesion improving [12], better geomechanical properties (both cohesion and friction angle) of rock mass [13-15], overall variation of rock mass properties [16, 17], adding internal pressure as a confining pressure within a broken rock mass [18-24], and stiffness improving of rock mass [7, 25]. Some of these researches are axisymmetric models on the basis of Convergence Confinement method which can be helpful to assess rock bolting effects in terms of tunnel convergence reduction [14, 15, 17, 18, 20].

In general, the previous models have proved that the bolting effect can be simulated with a rock mass properties variation but these models have rigorous

simplifications. On the other hand, most of the previously described axisymmetric models have been developed for either passively grouted rock bolts or ungrouted tensioned rock bolts [5, 24]

Therefore, a new theoretical model is presented based on convergence-confinement method for the active grouted rockbolts in tunneling design in which the effect of distance of bolted section on tunnel face is also considered. In this solution, it is assumed that the pre-tensioning of bolts develops pressure within the broken rock mass. Therefore, because of much more tension of active grouted rockbolts, internal pressure within a broken rock mass will be increased much more than the passive types.

## 2. Problem Definition

### 2.1. General Assumptions

A deep circular tunnel of radius  $r_i$  is driven in a homogeneous, isotropic, initially elastic rock mass subjected to a hydrostatic stress field  $P_0$  ( $K_0 = 1$ ) (see Fig. 3). The problem is studied in plane-strain conditions; therefore the three-dimensional effect even near the tunnel face is disregarded. The influence of the weight of the rock in the plastic zone on tunnel displacements is not considered and time-dependent changes properties of rock mass are neglected. Because of axial symmetry of the problem, the tangential and radial stresses,  $\sigma_\theta$  and  $\sigma_r$  in the rock mass surrounding the tunnel will be principal stresses,  $\sigma_1$  and  $\sigma_3$ , respectively.

Before rockbolts installation, it is assumed that the advancement of working face of the tunnel is as much as the stress induced in the rock at section where its distance from tunnel face is  $x$ , exceeds the yield strength of rock mass and an initial plastic zone of radius  $\bar{r}_e$  develops around the tunnel. It should be noted that developing plastic radius prior to support installation is proper assumption in most conditions [25].

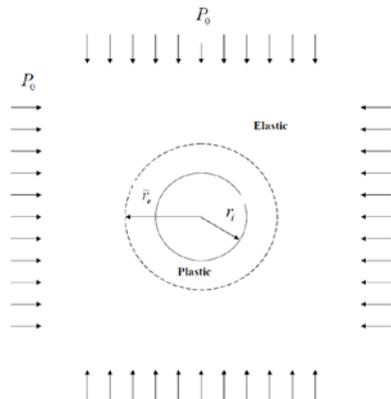


Fig. 3. The plane strain axisymmetric circular tunnel problem

### 2.2. Stress Analysis

The idealized stress-strain relationships used in the following analysis is elasto-strain softening as shown in Fig 4. (the compressive stress and related strain are considered positive). According to Hoek & Brown [26], under the usual condition (such as usual depth and GSI), this material behavior is proper for most of rock masses. The strength criterion adopted is that proposed in [5]:

$$\sigma_1 = \sigma_3 + (m \cdot \sigma_c \cdot \sigma_3 + s \cdot \sigma_c^2)^{1/2} \quad (1)$$

Where  $\sigma_1$  and  $\sigma_3$  are the major principal stress and minor principal stress at failure, respectively.  $\sigma_c$  is uniaxial compressive strength of the intact rock material, and parameters  $m$ ,  $s$  are rock mass constants depending on the nature of the rock mass and its geotechnical conditions before failure.

After failure, the strength criterions in the softening and the residual zones are [27]:

$$\sigma_1 = \sigma_3 + (\bar{m} \cdot \sigma_c \cdot \sigma_3 + \bar{s} \cdot \sigma_c^2)^{1/2} \quad (2)$$

$$\sigma_1 = \sigma_3 + (m_r \cdot \sigma_c \cdot \sigma_3 + s_r \cdot \sigma_c^2)^{1/2} \quad (3)$$

Where  $\bar{m}$ ,  $\bar{s}$  and  $s_r$ ,  $m_r$  are material constants in the softening and the residual zones.

It is assumed that both  $m$ ,  $s$  decrease from the peak values to the residual values  $m_r$ ,  $s_r$  linearly [27]. Therefore:

$$\bar{m} = m - (m - m_r) \frac{\eta}{\eta'} \quad (4)$$

$$\bar{s} = s - (s - s_r) \frac{\eta}{\eta'} \quad (5)$$

Where  $\eta$  and  $\eta'$  are strain softening and critical strain softening parameters. These parameters can be calculated from [28]:

$$\eta = \varepsilon_\theta - \varepsilon_{\theta e} \quad (6)$$

$$\eta' = (\alpha - 1) \varepsilon_{\theta e} \quad (7)$$

where  $f$  and  $h$  parameters describe the magnitudes of plastic behavior after the peak condition (ratio between the radial and tangential plastic strain both in the residual and softening branch of the stress-strain curve),  $\alpha$  defines the width of the softening zone (see Fig 4) in brittle material  $\alpha = 1$  and  $\varepsilon_{\theta e}$  is elastic strain corresponding to peak strength [26]:

$$\varepsilon_{\theta e} = \frac{M \sigma_c}{2G} \quad (8)$$

Where  $G$  the shear modulus of rock is mass and  $M$  is [27]:

$$M = \frac{1}{2} \left[ \left( \frac{m}{4} \right)^2 + m \frac{p_0}{\sigma_c} + s \right]^{\frac{1}{2}} - \frac{m}{8} \quad (9)$$

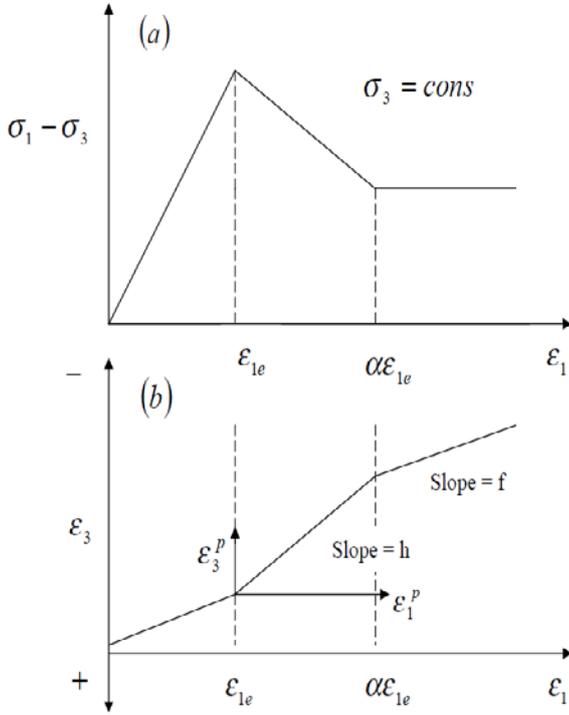


Fig. 4. Strain softening material behavior [27]

### 2.3. Strain and Displacement Analysis

Under the axisymmetric plane strain condition, the strains and the displacements are expressed as [27]:

$$\begin{aligned} \varepsilon_r &= \frac{du_r}{dr} \\ \varepsilon_\theta &= \frac{u_r}{r} \\ \varepsilon_z &= 0 \end{aligned} \quad (10)$$

Where  $\varepsilon_r$ ,  $\varepsilon_\theta$  and  $\varepsilon_z$  are radial, tangential and longitudinal strains, respectively and  $u_r$  is radial displacement at radius  $r$  from tunnel center.

The plastic strains in the plastic zone are governed by an appropriate flow rule postulated for the yielding behavior. Since the extent of yielding depends on the dilation characteristics of the failed rock, the flow rule must adopt the influence of dilation. In the present

solution, a linear Mohr-Coulomb plastic potential has been adopted. For an isotropic material, the principal axes of stress and strain increment coincide; therefore, under a plane strain condition, the ratio of the plastic strain increments can be given by:

$$d\varepsilon_r^p + N_\psi d\varepsilon_\theta^p = 0 \quad (11)$$

In which:

$$N_\psi = \frac{1 + \sin \psi}{1 - \sin \psi} \quad (12)$$

Where  $d\varepsilon_r^p$  and  $d\varepsilon_\theta^p$  are radial and plastic strain incremental and  $\psi$  is dilatancy angle.

Brown et al. [27] assumed that  $\psi$  in the softening zone is constant. On the other hand, Hoek and Brown [26] proposed  $\psi = \phi/8$  for softening material behavior.

## 3. Studies on Active Grouted Rockbolts in Tunnels

### 3.1. General Assumptions

- An axisymmetric bolt pattern consisting of identical active or passive grouted bolts are installed with  $S_c$  spacing around the circumference and with  $S_l$  spacing along longitudinal axis of tunnel (Fig 5).
- Grouted rockbolts are installed at a certain distance from tunnel face (refer to 2.1) (Fig 5).
- The length of a rockbolt should also be chosen so that it is anchored beyond the boundary of the broken zone in the original rock mass.
- The connection between bolt, grout and rock is rigid. Therefore, this paper does not address the shear displacement between the bolt and rock that may affect the axial stress distribution along the bolt.

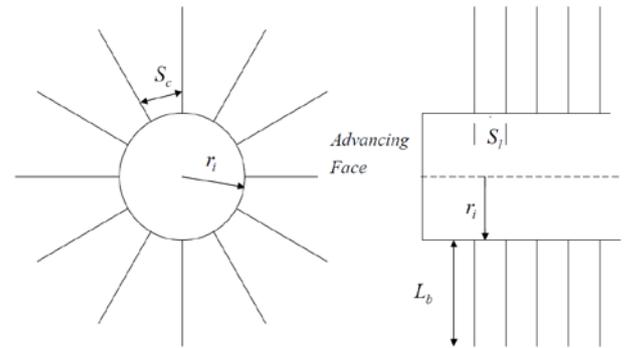


Fig. 5. Rockbolts arrangement

### 3.2. Modeling of Active and Passive Grouted Rockbolts

The analytical solutions that have been proposed for tunnel design can be classified into two groups: (1)

solutions that are derived based on the assumption of an "equivalent" material or homogenization method and (2) solutions obtained with a "smeared" approach.

In the first group, the properties of the medium surrounding the tunnel are those of a composite that includes both the rockbolts and the rock. The solution is found based on the engineering properties of the equivalent material [7, 11- 15, 18, 20, 29].

In the second group, the smeared approach, the contribution of the rockbolt is distributed to the rock surrounding the rockbolt. In this method, it is assumed that the tensile load of the rockbolt  $T$  introduces a radial compression in the rock with magnitude  $T/(S_l S_c)$  [16, 19, 21- 24, 30].

In this paper, through combining both approaches, the analytical model is proposed for passive and active grouted rockbolts. According to the "smeared" method, the radial stress in the reinforced rock mass is:

$$\sigma'_r = \sigma_r - \frac{T}{C} \quad (13)$$

$$C = S_l S_c \quad (14)$$

In which  $\sigma'_r$  is the adjusted radial stress.

On the other hand, the combination of rockbolts and rock mass can be replaced by composite medium whose strength is much more than rock mass strength due to having greater radial (confinement) stress ( $\sigma'_r$ ). (Note: The greater the radial stress, the greater the strength of rock mass)

The strength criterion of composite medium is:

$$\sigma_\theta - \sigma'_r = [m_r \sigma_c \sigma'_r + s_r \sigma_c^2]^{1/2} \quad (15)$$

Combination of Eqs (13) and (15) gives:

$$\sigma_\theta - \sigma_r = [m_r \sigma_c (\sigma_r - T/C) + s_r \sigma_c^2]^{1/2} - T/C \quad (16)$$

The differential equation of equilibrium is given by [27]:

$$\frac{d\sigma_r}{dr} = \frac{\sigma_\theta - \sigma_r}{r} \quad (17)$$

$$\frac{\sigma_{r(j-1)} - \sigma_{r(j)}}{r_{(j-1)} - r_{(j)}} = \frac{\left[ \frac{m_a \sigma_c}{2} \left[ (\sigma_{r(j)} + \sigma_{r(j-1)}) - \frac{T_{(j)} + T_{(j-1)}}{C} \right] + s_a \sigma_c^2 \right]^{1/2}}{\frac{r_{(j)} + r_{(j-1)}}{2}} - \frac{T_{(j)} + T_{(j-1)}}{2C} \quad (19)$$

Where  $\sigma_r$  and  $\sigma_\theta$  are radial and tangential stresses, respectively, at a point around the tunnel at a radius  $r$  from the tunnel centre.

Substituting Eq. (16) into Eq. (17) leads to equilibrium equation of composite medium:

$$\frac{d\sigma_r}{dr} = \frac{[m_r \sigma_c (\sigma_r - T/C) + s_r \sigma_c^2]^{1/2} - T/C}{r} \quad (18)$$

In the strain softening zone  $m_r$  and  $s_r$  are substituted by  $\bar{m}$  and  $\bar{s}$ . In this situation, the Esq. will be very complex and do not have closed form solution. Therefore, a numerical solution using a simple stepwise procedure based on the finite difference method is adopted to solve the differential equations of equilibrium and distribute stresses and strains in the plastic zone. The stepwise solution involves the discretization of the plastic zone in annular rings starting from the unknown elastic-plastic interface towards the tunnel by incrementing the tangential strain value  $\varepsilon_{\theta(j)}$  for every calculation step (for example  $0.01\varepsilon_{\theta(j)}$ ) and calculating the corresponding radial strain  $\varepsilon_{r(j)}$  (the meshing and splitting of the medium is shown in Fig. 6).

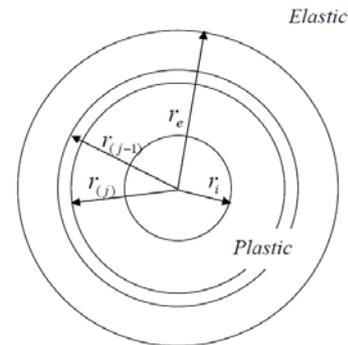


Fig. 6. Typical annulus in plastic zone used in step wise solution

Now, Using Eq. (18) for a general thin annular ring between  $r_{(j-1)}$  and  $r_{(j)}$ , it may be written as an approximation:

$m_a$  And  $s_a$  material properties for ring  $r_{(j)}$ . The bolt load is:

$$T = A_b \cdot E_s \cdot \varepsilon_t \quad (20)$$

Where  $A_b$  cross section area of bolt is,  $E_s$  is elasticity modulus of bolt and  $\varepsilon_b$  is total bolt strain.

In active grouted rockbolts, axial tension load is developed not only by shear stress transmission due to rock mass deformation along the bolt shank (like passive grouted bolts), but also is produced by applying the pretension load. Pre-tension bolt load is:

$$T_{pre} = A_b \cdot E_s \cdot \varepsilon_{pre} \quad (21)$$

and corresponding average radial stress in rock mass is:

$$\sigma_{pre} = \frac{T_{pre}}{C} \quad (22)$$

Where  $\varepsilon_{pre}$  is pre-tension strain. According to the static equilibrium,  $T_{pre}$  is constant along the bolt, but  $C$  is not constant along the bolt. (Influence area of bolt decreased due to increasing  $S_c$  along the bolt). However, for the sake of simplicity, it is assumed,  $C$  is constant and then  $\sigma_{pre}$  is also constant along the bolt.

After grouting the remainder of the bolt length and advancing the working face, the plastic zone becomes greater (see fig 7) and the whole broken rock mass moves into the tunnel and interacts with rock bolts. Therefore, total axial tension load in active grouted rockbolts is given by:

$$T = A_b E_s (\varepsilon'_r + \varepsilon_{pre}) \quad (23)$$

where  $\varepsilon'_r$  is the radial strain within rock mass that takes place after rockbolt installation. The radial strain of rock mass:

$$\varepsilon'_r = \begin{cases} \varepsilon_r - \bar{\varepsilon}_r & r_i \leq r < \bar{r}_e \\ \varepsilon_r - \varepsilon_{re} & \bar{r}_e \leq r \leq r_e \end{cases} \quad (24)$$

where  $\varepsilon_r$  is total radial strain of plastic zone at  $r$  (can be calculated from Brown et al [27] method which briefly explained in section 3.2),  $\bar{\varepsilon}_r$  is the radial strain in initial plastic zone that take place before rockbolt installation at

$r$  (can be calculated like  $\varepsilon_r$  by [27]),  $\bar{r}_e$  and  $r_e$  is initial and final plastic radius.

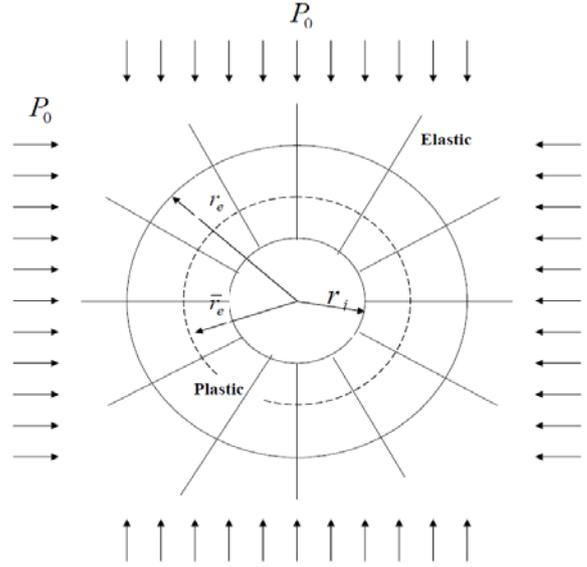


Fig 7. Increasing plastic radius after rockbolt installation and advancing working face

The active rockbolt axial force for the annular located in the initial plastic zone (which is produced before rock bolt installation) can be obtained as:

$$T_{(j)} = A_b E_s (\varepsilon_{r(j)} - \bar{\varepsilon}_{r(j)} + \varepsilon_{pre}) \quad (25)$$

$$r_i \leq r < \bar{r}_e$$

and for the annular ring located in the plastic zone which is being produced in excavation process after rockbolts installation can be approximately obtained as:

$$T_{(j)} = A_b E_s (\varepsilon_{r(j)} - \varepsilon_{re} + \varepsilon_{pre}) \quad (26)$$

$$\bar{r}_e \leq r < r_e$$

Firstly, solving of the problem would be continued for the  $r_i \leq r < \bar{r}_e$  condition.

Combination of Eqs. (19) and (25) gives:

$$\frac{\sigma_{r(j-1)} - \sigma_{r(j)}}{r_{(j-1)} - r_{(j)}} = \frac{\left[ \frac{m_a \sigma_c}{2} \left[ (\sigma_{r(j)} + \sigma_{r(j-1)}) - \frac{A_b E_s}{C} (\varepsilon_{r(j)} + \varepsilon_{r(j-1)} - \bar{\varepsilon}_{r(j)} - \bar{\varepsilon}_{r(j-1)} + 2\varepsilon_{pre}) \right] + s_a \sigma_c^2 \right]^{1/2}}{\frac{r_{(j)} + r_{(j-1)}}{2}} \quad (27)$$

$$- \frac{A_b E_s (\varepsilon_{r(j)} + \varepsilon_{r(j-1)} - \bar{\varepsilon}_{r(j)} - \bar{\varepsilon}_{r(j-1)} + 2\varepsilon_{pre})}{C (r_{(j)} + r_{(j-1)})}$$

For a very thin annular ring and according to the relations between displacements and strains, it can be written as [27]:

$$\frac{r_{(j)}}{r_{(j-1)}} = \frac{2\varepsilon_{\theta(j-1)} - \varepsilon_{r(j-1)} - \varepsilon_{r(j)}}{2\varepsilon_{\theta(j)} - \varepsilon_{r(j-1)} - \varepsilon_{r(j)}} \quad (28)$$

The following parameters are also defined as: [27]

$$\lambda_{(j)} = \frac{r_{(j)}}{r_e} \quad (29)$$

$$\bar{\lambda}_{(j)} = \frac{r_{(j)}}{\bar{r}_e} \quad (30)$$

Combination of Eqs. (28) and (29) gives [27]:

$$\beta = \frac{\lambda_{(j)}}{\lambda_{(j-1)}} = \frac{2\varepsilon_{\theta(j-1)} - \varepsilon_{r(j-1)} - \varepsilon_{r(j)}}{2\varepsilon_{\theta(j)} - \varepsilon_{r(j-1)} - \varepsilon_{r(j)}} \quad (31)$$

And also with definition of [27]:

$$\gamma = \varepsilon_{r(j-1)} + \varepsilon_{r(j)} = \frac{2[\lambda_{(j-1)}\varepsilon_{\theta(j-1)} - \lambda_{(j)}\varepsilon_{\theta(j)}]}{\lambda_{(j-1)} - \lambda_{(j)}} \quad (32)$$

$$\bar{\gamma} = \bar{\varepsilon}_{r(j-1)} + \bar{\varepsilon}_{r(j)} = \frac{2[\bar{\lambda}_{(j-1)}\bar{\varepsilon}_{\theta(j-1)} - \bar{\lambda}_{(j)}\bar{\varepsilon}_{\theta(j)}]}{\bar{\lambda}_{(j-1)} - \bar{\lambda}_{(j)}} \quad (33)$$

$$\gamma_{pre} = 2\varepsilon_{pre} \quad (34)$$

(Note: subscripts ( $j$ ) and ( $j-1$ ) refer to rings at plastic zone at radius  $r_{(j)}$  and ( $j-1$ ))

For abbreviation it can be written: [27]

$$K = \frac{\lambda_{(j-1)} - \lambda_{(j)}}{\lambda_{(j)} + \lambda_{(j-1)}} \quad (35)$$

$$K_1 = \frac{A_b E_s}{2C} \gamma \quad (36)$$

$$\bar{K}_1 = \frac{A_b E_s}{2C} \bar{\gamma} \quad (37)$$

$$K_2 = \frac{m_a \sigma_c}{4} \quad (38)$$

$$K_t = \frac{A_b E_s}{2C} \gamma_{pre} \quad (39)$$

Combination of Eqs. (32) – (34) and (27) and then substitution of Eqs (35)-(39) gives the second order equation for  $\sigma_{r(j)}$ :

$$a \cdot \sigma_{r(j)}^2 + b \cdot \sigma_{r(j)} + c = 0 \quad (40)$$

In which:

$$a = \frac{1}{4K^2}$$

$$b = \frac{\bar{K}_1 - K_1 - K_t}{K} - \frac{\sigma_{r(j-1)}}{2K^2} - 2K_2$$

$$c = \sigma_{r(j-1)} \left[ \frac{\sigma_{r(j-1)}}{4K^2} + \frac{K_1 - \bar{K}_1}{K_2} - 2K_2 \right] + (K_1 - \bar{K}_1)^2 + 4K_2(K_1 - \bar{K}_1) - s_a \sigma_c^2 \quad (41)$$

For  $\bar{r}_e \leq r < r_e$  region and considering Eq. (26), and then replacing  $\bar{K}_1$  by  $K_e$ :

$$K_e = \frac{A_b E_s}{2C} \gamma_e \quad (42)$$

$$\gamma_e = 2\varepsilon_{re} = -\frac{M\sigma_c}{G} \quad (43)$$

Gives the second order equation for  $\sigma_{r(j)}$  to that region.

The boundary conditions for these equations are, respectively, defined as follows:

- (I) At  $r = r_i$  ;  $\sigma_r = p_i$  and
- (II)  $r = r_e$  ;  $\sigma_r = \sigma_{re}$

$\sigma_{re}$  is radial stress in plastic- elastic interface and can be calculated by:

$$\sigma_{re} = p_0 - M\sigma_c \quad (44)$$

For further information about the boundary condition and solving the Eqs, refer to Appendix B in [27].

#### 4. Examples

Some examples from [27] are chosen and ground response curve calculations, for a rock mass being reinforced by grouted rockbolts, are performed and the corresponding curve is plotted. In this method, the effect of active and passive grouted rockbolts in confining tunnel convergence is quantified.

In these examples, the influence of some parameters of the model on the reinforced tunnel convergence is also investigated:

- The distance of unlined section to tunnel face.
- The bolts spacing.
- The magnitude of pretension load.

**Example 1:**

A circular mine haulage tunnel of radius 4m, is to be driven in a good quality quartzite at a depth at which the in situ hydrostatic stress is  $p_0 = 81 \text{ MPa}$ . The rock mass properties are

$$\sigma_c = 300 \text{ MPa}, m = 7.5, s = 0.1, E = 40 \text{ GPa}, m_r = 0.3, s_r = 0.001, f = 1.$$

In this example  $A_b$  (rockbolt cross-section area) has been assumed equal to zero ( $A_b = 0$ ). Thus, it is expected that the corresponding results are the same as the results obtained by Brown et al. (1983).

Brown et al. [27] reported that the ratio  $\frac{u_i}{r_i}$  for  $\frac{P_i}{P_0} = 0$

will be 0.450% for  $p_0 = 81 \text{ MPa}$ . This ratio has been calculated 0.443%, which shows very good agreement with the value reported by [27].

**Example 2:**

A highway tunnel with diameter 10.7 m is driven in a fair to good quality limestone at a depth of 122 m below the surface. The following material properties are given for the rock mass

$$m = 0.5, f = 1.2, h = 2, \alpha = 3.5, \text{MPa } \sigma_c = 27.6, \nu = 0.25 \text{ GPa } E_r = 4.38, \text{MPa } P_0 = 3.31, s = 0.001, s_r = 0, m_r = 0.1,$$

The output results and the related response curve are shown in Fig. 8. According to [27] report, the ratio  $\frac{u_i}{r_i}$  for

$\frac{P_i}{P_0} = 0$  will be 1.65% which shows relatively good agreement with the results obtained by theoretical method (calculated 1.48%).

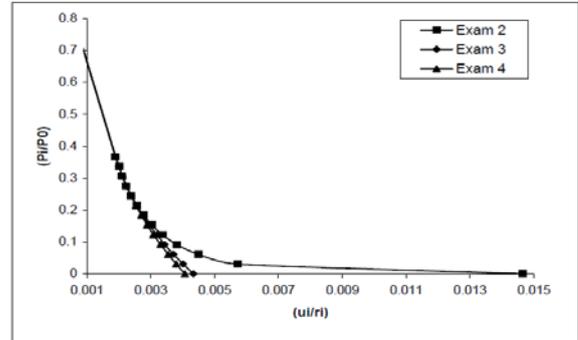


Fig. 8. Ground response curve for the rock mass around the tunnel in Examples 2-4

Following Examples 3-7 with the same rock mass input data of Example 2 (but different bolting design input data) were solved to study and investigate the influence of different design parameters involved in the mathematical formulation of the proposed model on the tunnel convergence. Bolting design input data of these examples are illustrated in Table 1.

Examples 3 and 4 discussed the existence of passive and active grouted rockbolts (reinforced elements) in confining the amount of deformation in plastic zone and compare with unreinforced rock mass. In Examples 5-8, the following grouted bolting design parameters were investigated:

- The distance of unlined section to tunnel faces (Example 5).
- The bolts spacing (Example 6).
- The magnitude of pretension load (Example 7).

Table 1: The input data

| Examples | Cross section area<br>(cm <sup>2</sup> ) | Pre-tension load<br>(t) | C<br>(m <sup>2</sup> ) | Distance from tunnel face<br>(m) |
|----------|--|-------------------------|------------------------|----------------------------------|
| Exam 2   | 0  | -                       | -                      | -                                |
| Exam 3   | 5  | 0                       | 0.5                    | 2                                |
| Exam 4   | 5  | 8                       | 0.5                    | 2                                |
| Exam 5A  | 5  | 8                       | 0.5                    | 2.5                              |
| Exam 5B  | 5  | 8                       | 0.5                    | 1.5                              |
| Exam 6A  | 5  | 8                       | 0.75                   | 2                                |
| Exam 6B  | 5  | 8                       | 0.25                   | 2                                |
| Exam 7A  | 5  | 12                      | 0.5                    | 2                                |
| Exam 7B  | 5  | 4                       | 0.5                    | 2                                |

4.1. The Results of Analyses and Examples

As observed in Fig. 8 (the ground response curve of Examples 2–4), both passive and active grouted rockbolts decrease deformation in the plastic zone considerably in comparison to Example 2. In Example 4, it can also be perceived the amount of displacements in plastic zone decrease in comparison to Example 3. In fact, by applying the pretension load the initial and total radial internal pressure of the rock mass as the confining pressure increases with respect to passive types.

In all examples except Example 5, the grouted rockbolts were installed in distance of 2 m from working face, the corresponding fictitious radial internal pressure in that section due to confinement of tunnel face is about

$$p_i = 0.81 \text{ MPa} \text{ (or } \frac{P_i}{P_0} = 0.248\text{)}. \text{ For 5A and 5B}$$

examples, the corresponding fictitious radial internal

$$\text{pressure are } \frac{P_i}{P_0} = 0.173 \text{ and } \frac{P_i}{P_0} = 0.367 .$$

The ground response curve (GRC) graphs of Examples 4, 5A and 5B are shown in Fig. 9

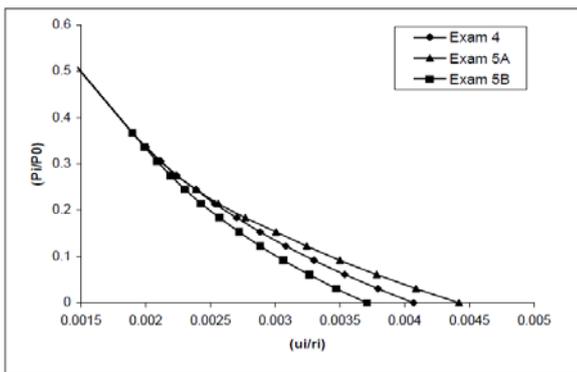


Fig. 9. Ground response curve for the rock mass around the tunnel in Examples 2, 5A and 5B.

As observed above, time of rockbolts installation is a very important factor in tunneling design. Therefore, as many studies have stated and this analytical model shows, the rockbolts (or any type of support system) should be installed after excavation as soon as possible or the grouted rockbolts may not have the proper efficiency as a support system.

Example 6 investigates the influence of the installation spacing of grouted rockbolts. Since varying the number of rockbolts per unit area of the tunnel surface is corresponding to increase or decrease of the internal pressure induced in the broken rock mass, it influences the tunnel convergence. The GRC graphs of Examples 4, 6A and 6B are shown in Fig. 10 which reflects the sensitivity of tunnel convergence to bolting pattern installation.

In Examples 7A and 7B, the pretension load of bolts was different. As observed in Fig 11, the amount of displacements in the reinforced plastic zone is dependent

upon the magnitude of the pretension load. Due to increasing of the pretension load, the initial and total radial internal pressures within the rock mass as the confining pressure increases.

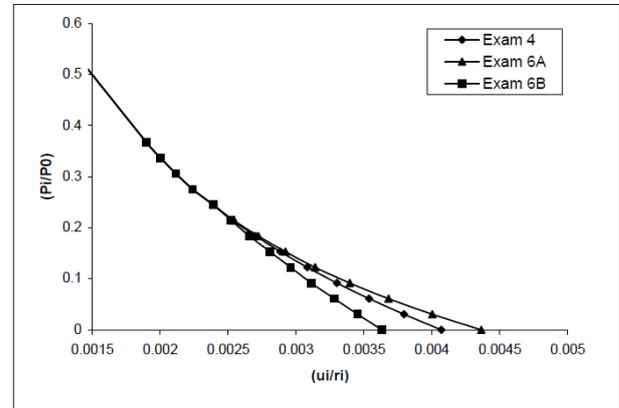


Fig. 10. Ground response curve for the rock mass around the tunnel in Examples 2, 6A and 6B.

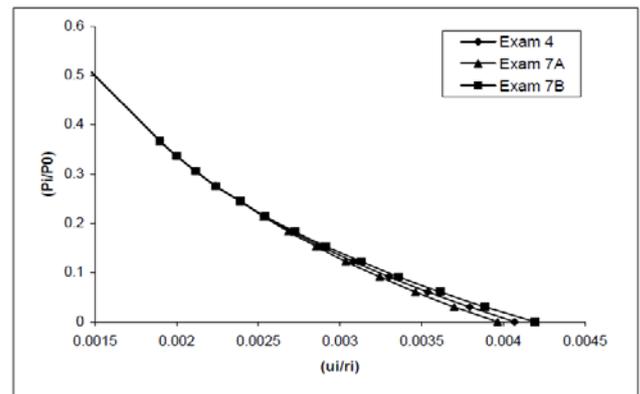


Fig. 11. Ground response curve for the rock mass around the tunnel in Examples 2, 7A and 7B.

5. Summary and Conclusions

An analytical solution is proposed for the computation of the ground reaction curve of a tunnel which was reinforced with active grouted rockbolts while the effect of the distance of the bolted section to tunnel face is considered. It is extended for strain softening behaviors and non-linear peak strength criterion. Different examples are solved using this method and the results obtained are comparable to the literature and confirm the correctness and suitability of the method.

The results show that preloading the bolts is not as effective as decreasing the bolts spacing in confining tunnel convergence.

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